Multiscale multifractality analysis of a 12-lead electrocardiogram

Jun Wang,[1] Xinbao Ning,[1,*] Qianli Ma,[1] Chunhua Bian,[1] Yinlin Xu,[1,2] and Ying Chen[1]
[1]*State Key Laboratory of Modern Acoustics, Department of Electronic Science and Engineering, Institute for Biomedical Electronic Engineering, Nanjing University, Nanjing 210093, People's Republic of China*
[2]*College of Physics Science and Technology, Nanjing Normal University, Nanjing 210097, China*
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This paper proposes that a multiscale multifractality (MSMF) method be adopted for the spatiotemporal analysis of 12-lead ECG. By using this method, the authors find that, in some frequency range, 12-lead ECG has a more complex fractal structure, and the position of the largest singularity strength range $\Delta\alpha$ is not relying on the data length but on the scale factor. By determining the inflexion, the MSMF proves to be more sensitive in displaying the trend that the singularity strength range $\Delta\alpha$ of human ECG decreases with human aging.

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The report of multifractal structure [1] has led to the application of it in a number of fields such as condensed matter [2], heartbeat signal analysis [3–6], etc. There is evidence that physiological signals generated by complex self-regulating systems may have a fractal structure. Ivanov *et al.* [4] reported that time series of healthy human interbeat intervals belong to a special class of complex signals that display multifractal properties. Ivanov *et al.* [4] and Amaral *et al.* [6] both reported that the multifractal properties of heart rate variability of the human body were mainly under the control of the neuroautonomic system.

Wang *et al.* reported that the mean value of the areas of multifractal singularity spectrum for 12-lead ECG of a human is mainly controlled by the strength of the body's neuroautonomic control on the heart, but not the extent of heart disease [7]. This research echoes the assumption of Ivanov *et al.* [4] and Amaral *et al.* [6]. The synchronous 12-lead ECG multifractal singularity spectrum distribution is modulated by the heart disease information [8].

Zhang [9] proposed a general method to measure the multiple time scales in physical systems; based on Zhang's research, Costa *et al.* [10] proposed the multiscale entropy (MSE) method. Stimulated by these two researches, the authors in this paper propose a multiscale multifractality (MSMF) method for the spatiotemporal analysis of physiologic time series. The multifractality method analyzes the dimensions of time series in different time segments but misses the information in different time scaling domains.

Traditional analysis of physiological time series includes many methods such as correlation dimension [11–13], Lyapunov exponents [14], approximate entropy [15,16], sample entropy [17], mode entropy [18], multiscale entropy [19], multifractality [1,4,6], etc. In our understanding, any single one of the parameters referred to in the abovementioned methods may be insufficient in revealing the hidden information of the researched physiological and pathological signals. The synthetic multiparameter presented in proposed theories by many researchers can help to further understanding of the inner information of the physiological and pathological system.

Given a one-dimensional discrete time series, $\{x_1,\ldots,x_i,\ldots,x_N\}$, we construct the consecutive coarse-grained time series of it, that is $\{y^{(\tau)}\}$,

$$y_j^{(\tau)} = \frac{1}{\tau}\sum_{i=(j-1)\tau+1}^{j\tau} x_i, \quad 1 \leqslant j \leqslant N/\tau, \tag{1}$$

where τ is the scale factor.

The length of each coarse-grained time series is equal to the length of the original time series divided by the scale factor τ [9,10]. In a case where $\tau=1$, the time series $\{y^{(1)}\}$ is simply equal to the original time series.

The present research takes into account the case in which 55 440 points are primarily chosen as the time series where the coarse-grained factor varies from 1 to 10, and thus calculates the multifractality measure for each coarse-grained time series [1,20]. This procedure is termed multiscale multifractality (MSMF) analysis and is used to produce the $f(\alpha)$ [1] spectrum proposed by Chhabra and Jensen [20].

The measure chain is covered with segments of size L and the probability $P_i(L)$ is calculated in each of these segments. The multifractal formalism accounts for the statistical properties of some measure in terms of its distribution of the singularity spectrum $f(\alpha)$ corresponding to its singularity strength α.

In this research, probability P_i is determined by:

$$P_i = T_i \Big/ \sum_{i=1}^{N} T_i, \tag{2}$$

where T_i is the mean of the ith segment. When the length of the chain is divided into N equal small segments it can be calculated by summing the measure in the ith segment.

The normalized qth moment of the probability measure P_i is determined by the following expression:

$$\mu_i(q,L) = [P_i(L)]^q \Big/ \sum_{j=1}^{N} [P_i(L)]^q, \tag{3}$$

where L is equal to N^{-1}.

*Author to whom correspondence should be addressed; electronic mail: xbning@nju.edu.cn

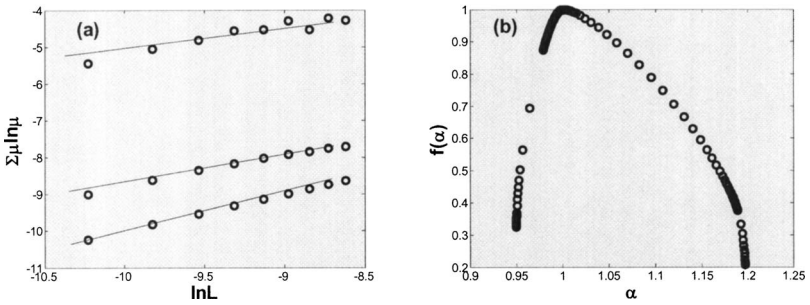

FIG. 1. (a) $\Sigma\mu_i \ln \mu_i$ vs $\ln L$ with q from the top in the order of $-10, -5$, 0.5. (b) $f(\alpha)$vs α shows multifractal singularity spectrum of human ECG signal.

The Hausdorff dimension of the measure theoretic support of $\mu(q)$ is given by:

$$f(q) = -\lim_{N\to\infty} \frac{1}{\ln N} \sum_{i=1}^{N} \mu_i(q,L) \ln[\mu_i(q,L)]$$

$$= \lim_{L\to 0} \frac{\sum \mu_i(q,L) \ln[\mu_i(q,L)]}{\ln L}. \quad (4)$$

In addition, the average value of the singularity strength $\alpha_i = \ln(P_i)/\ln L$ with respect to $\mu(q)$ can be determined by:

$$\alpha(q) = -\lim_{N\to\infty} \frac{1}{\ln N} \sum_{i=1}^{N} \mu_i(q,L) \ln[P_i(L)]$$

$$= \lim_{L\to 0} \frac{\sum \mu_i(q,L) \ln[P_i(L)]}{\ln L}. \quad (5)$$

Equations (4) and (5) indicate the relationship between a Hausdorff dimension f and an average singularity strength α; f and α both are functions of the parameter q. The singularity strength α functions as a scaling exponent and $f(\alpha)$ as the corresponding fractal dimension.

In our understanding, the multiscale multifractality (MSMF) method has the following significance: coarse graining the time series implies that the sampling frequency of the coarse-grained time series is changing; calculating the multifractality of the coarse-grained time series makes it possible to analyze the fractal dimensions in different time segments of that series.

Multiscale multifractality algorithm is tested by analyzing 65 ECG data sets, with the sampling frequency 1 kHz, taken from healthy human subjects in rest condition. All data sets utilize wavelet filtering (the wavelet function chosen is bior6.8) for removing respiration wave (less than 0.5 Hz) and 50 Hz noise.

Consider several time series with 55 440 points and coarse grain them up to 10; every coarse-grained time series is divided into many segments and each segment is comprised of just two points. The parameter q varies from $-\infty$ to ∞ and the step varies in need.

All the data are processed by MATLAB software. Figure 1(a) shows an example of the linear fit to $\Sigma\mu_i \ln \mu_i$ vs $\ln L$ with q from the top in the order of $-10, -5$, 0.5. It shows that there is no ambiguity in determining the slopes. The f value remains unchanged as the time series are segmented (provided that $L \to 0$). This confirms that the multifactal singularity spectrum [Fig. 1(b)] is reasonable [7,8]. Human ECG is characterized by multifractal structure [7,8], which shows the nonlinear complexity of heartbeat signals.

To verify this method, the authors use two-scale Cantor sets, which have been confirmed multifractality [20]. The Cantor sets are generated by dividing the unit interval into two pieces, each being half the previous length. This process is infinitely repeated [20] where the two halves are of different probabilities (say $p1$ and $p2$). In Fig. 2(a), $p1 = 0.7000$, $p2 = 0.3000$; in Fig. 2(b), $p1 = 0.6999$, $p2 = 0.3001$. In Figs. 2(a) and 2(b) 55 440 points are taken. The singularity strength range $\Delta\alpha$ is defined as the difference between the maximum $\alpha_{\max}$ and the minimum $\alpha_{\min}$. Figure 2 illustrates that the singularity strength range $\Delta\alpha$ is distributed along with scale factors τ. There also exist highest singularity strength range values in scale factor 3. This implies that the multiscale multifractal method is effective in denoting the scale factor, which shows the most complex fractal structure of the researched time series.

Then take healthy human 55 440-point ECG for the multiscale multifractality (MSMF) analysis. Figure 3 shows the results of three different subjects: the singularity strength range $\Delta\alpha$ reaches its maximum value when the scale factor τ equals 4 in Fig. 3(a), equals 5 in Fig. 3(b), and 6 in Fig. 3(c). It is found that when the scale factor is from 4 to 6, the singularity strength range $\Delta\alpha$ of 12-lead ECG is larger than

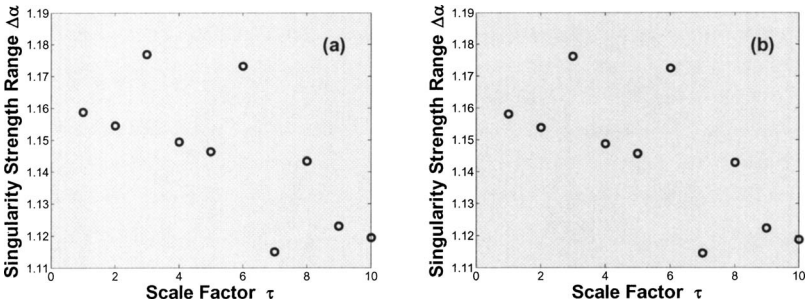

FIG. 2. Singularity strength range $\Delta\alpha$ vs scale factor τ [(a) $p1 = 0.7000$, $p2 = 0.3000$; (b) $p1 = 0.6999$, $p2 = 0.3001$].

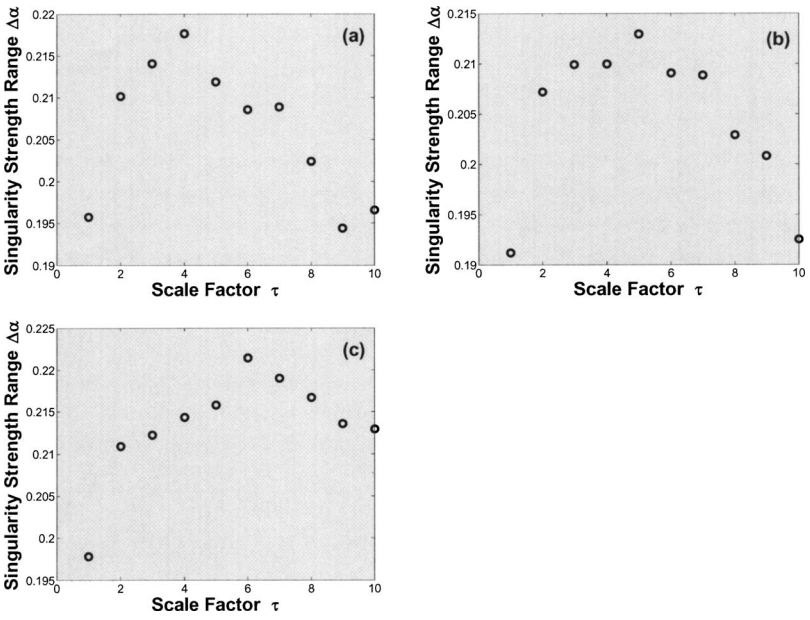

FIG. 3. Curves of the singularity strength range $\Delta\alpha$ vs scale factor τ for 55 440-point data [(a) τ=4, (b) τ=5, (c) τ=6].

that in other cases. This shows that the distribution of the singularity strength range $\Delta\alpha$ is higher in the middle but lower on both sides (Fig. 3).

Apply the MSMF method to 65 subjects with a 55 440-point ECG. The scale factor τ with the maximum singularity strength range $\Delta\alpha$ is from 3 to 8, the corresponding number is, respectively, 3, 15, 30, 10, 4, and 3. This implies that, in most cases, the scale factor τ is able to denote the maximum singularity strength range $\Delta\alpha$ when τ equals 4, 5, or 6. It also implies that the appropriate sampling frequency is very important for analyzing the multifractal structure of a 12-lead ECG. It is thus concluded that a 12-lead ECG has a more complex fractal structure in a certain frequency range.

All the above data were sampled with the frequency 1 kHz. The coarse-grained time series were reconstructed under different sampling frequencies (coarse-grained sampling frequency), which were obtained by the original sampling frequencies divided by the scale factor. When the scale factor is from 4 to 6, the corresponding coarse-grained sampling frequency varies from 250 to 166 Hz. While for the ECG, most sensitive physiologic information is obtained when coarse-grained sampling frequency is in the region of 250–166 Hz. This indicates that not all of the sampling frequency is applicable to analyzing the multifractal structure of physiologic time series. By reconstructing the coarse-grained time series, the frequency region, which most effectively expressing the multifractal structure of physiologic time series, can be determined.

To further confirm the above conclusion, the authors applied the MSMF method to three more sets of 40 320-point ECG data for the same subjects as mentioned in Fig. 3. They were coarse grained up to scale 10. The authors then calculated the singularity strength range $\Delta\alpha$ of the coarse-grained series in the same way as described above.

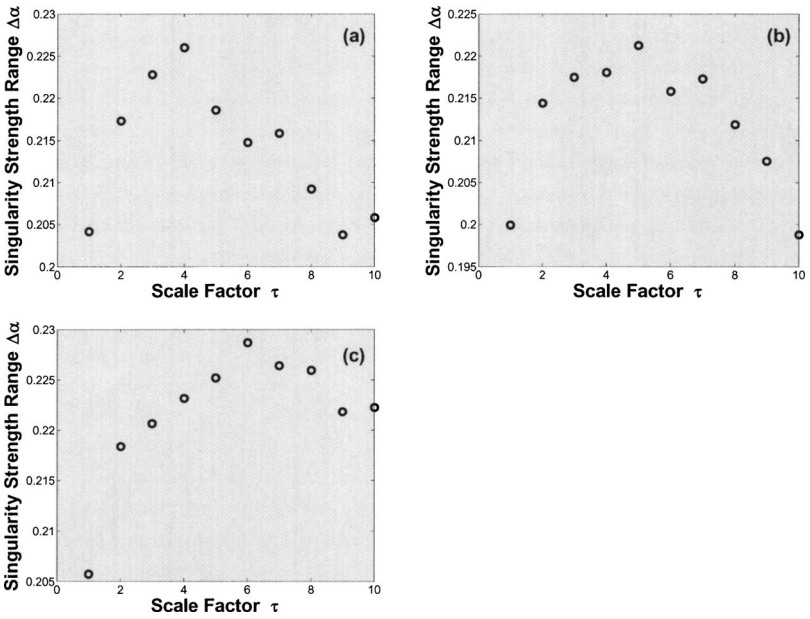

FIG. 4. Curves of the singularity strength range $\Delta\alpha$ vs scale factor τ for 40 320-point data [(a) τ=4, (b) τ=5, (c) τ=6].

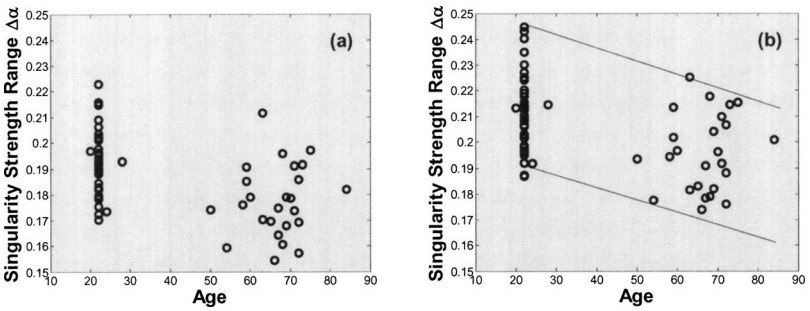

FIG. 5. Singularity strength range $\Delta\alpha$ of healthy human ECG varies according to age [age unit is year., (a) $\tau=1$; (b) $\tau=4$: In the (b), parallel is given to show the trend.]

Figure 4 shows that $\Delta\alpha$ has the same distribution as that in Fig. 3. Distribution is higher in the middle but lower in the both sides. Contrasting Figs. 4(a)–4(c) with Figs. 3(a)–3(c) further proves that the scale factors with the maximum singularity strength ranges $\Delta\alpha$ bears no relation with data length. This implies that for each 12-lead ECG data the maximum singularity strength range $\Delta\alpha$ is not relying on the data length but on the scale factor. This also proves the significance of the multiscale multifractality method.

Apply the MSMF method to the 55 440-point ECG data. It is reported that the mean value of the areas of the multifractal singularity spectrum for 12-lead ECG of a human is mainly controlled by the strength of the body's neuroautonomic control on the heart, but not the extent of heart disease [7]. This research echoes the assumption of Ivanov *et al.* [4] and Amaral *et al.* [6].

Applying the MSMF method to 65 healthy subjects with 55 440-point ECG and determining the relation between $\Delta\alpha$ and the age of the subjects was done by calculating the singularity strength range $\Delta\alpha$ of the above data. Figure 5 shows that the scale factor is more sensitive when $\tau=4$ [Fig. 5(b)] than $\tau=1$ [Fig. 5(a)] in accounting for the trend that the singularity strength range $\Delta\alpha$ of human ECG decreases as the tested subject ages. In the case where the scale factor is larger than 4, the decreasing trend is insignificant. This proves that the scale factor 4 is an important inflexion. This confirms that the determination of an appropriate sampling frequency is of vital importance to the analysis of the multifractality structure of a 12-lead ECG.

In Figs. 5(a) and 5(b), the linear regression method was adopted to examine the confidence intervals of the singularity strength ranges $\Delta\alpha$ of the 65 ECG data, to confirm that $\Delta\alpha$ is correlated with y_{age}.

In the case of $\tau=1$,

$$\Delta\alpha_{\tau 1} = a_{\tau 1} + b_{\tau 1} * y_{\text{age}} \tag{6}$$

where $a_{\tau 1}=0.2001$, $b_{\tau 1}=-0.0003$; and when the confidence is 0.95,

$$C_{a_{\tau 1}} = (0.1934, 0.2068),$$

$$C_{b_{\tau 1}} = (-0.0005, -0.0002).$$

In the case of $\tau=4$,

$$\Delta\alpha_{\tau 4} = a_{\tau 4} + b_{\tau 4} * y_{\text{age}} \tag{7}$$

where $a_{\tau 4}=0.2186$, $b_{\tau 4}=-0.0003$; and when the confidence is 0.95,

$$C_{a_{\tau 4}} = (0.2111, 0.2261),$$

$$C_{b_{\tau 4}} = (-0.0005, -0.0002).$$

This research establishes that the awareness of appropriate sampling frequency is of vital importance in analyzing the multifractal structure of a 12-lead ECG and that, in a certain frequency range, a 12-lead ECG has a more complex fractal structure. For a 12-lead ECG the position of the maximum singularity strength range $\Delta\alpha$ is not relying on the data length but on the scale factor. It is also found that by determining the inflexion, the MSMF proves to be more sensitive in displaying the trend that the singularity strength range $\Delta\alpha$ of an ECG of a human decreases with aging.

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